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Time-dependent reliability analysis of wind turbines considering load-sharing using fault tree analysis and Markov chains

Yao Li[1] and Frank P.A. Coolen[2]

Abstract

Due to the high failure rates and the high cost of operation and maintenance of wind turbines, not only manufacturers but also service providers try many ways to improve the reliability of some critical components and subsystems. In reality, redundancy design is commonly used to improve the reliability of critical components and subsystems. The load dependencies and failure dependencies among redundancy components and subsystems are crucial to the reliability assessment of wind turbines. However, the redundancy components are treated as a parallel system, and the load correlations among them are ignored in much literature, which may lead to the wrong system's reliability and much higher costs. For this reason, this article explores the influences of load-sharing on system reliability. The whole system's reliability is quantitatively evaluated using fault tree analysis and the Markov chain method. Following this, the optimisation of the redundancy allocation problem considering the load-sharing is conducted to maximise the system reliability and reduce the total cost of the system subjecting to the available system cost and space. The results produced by this methodology can show a realistic reliability assessment of the whole wind turbine from a quantitative point of view. The realistic reliability assessment can help to design a cost-effective and more reliable system and significantly reduce the cost of wind turbines.

Keywords

wind turbines, reliability assessment, load-sharing, fault tree analysis, Markov chains

Introduction

Fossil fuels are non-renewable energy sources and have a severe impact on the environment. Developing renewable energy sources is a crucial measure to help achieve energy security and sustainable development. The increasing environmental and climatic concerns of current times have moved the research focus from conventional electricity sources to renewable energy. Currently, wind energy is one of the most promising source of energy across the world.

The wind turbine is a typical example of mechatronics systems equipped with high technologies, which can capture the kinetic energy of wind and transform it into electric energy. The operating condition of wind turbines (WTs) is variable. The reliability and safety of WTs are influenced by many factors, such as random wind speed, high temperature and sand. With an increasing number of wind turbines being installed, lots of potential problems still need to be solved, such as structural fatigue, high failures and low reliability. This is especially true in the current circumstances where tower height, rotor diameter, and overall turbine weights have almost quadrupled in size and capacity.[1,2] The above reasons have led to more accidents. According to accident statistics from Caithness Windfarm Information Forum, more and more accidents are occurring with an average of 33 accidents per year from 1998-2002; 81 accidents per year from 2003-2007 inclusive; 144 accidents per year from 2008-2012 inclusive, and 167 accidents per year from 2013-2017 inclusive.[3,4] High failure rates will lead to the high cost of operation and maintenance (O&M), which may reduce wind farm profits. The O&M costs of onshore wind turbines account for about 10%-15% of wind farm income in a 20-year design life. For an offshore wind turbine, O&M costs are as high as 20%-25% of wind farm income.[5] Because the warranty period for most older wind turbines has expired, a large amount of WTs suffer high failure rates that causes high O&M costs. Therefore, alike investors, operators and service providers, all want to improve wind turbine reliability and minimise wind turbine O&M costs.

It has been argued that larger wind turbines tend to fail more frequently than smaller ones.[1] Because of this challenge in reality, improving WT reliability is becoming increasingly necessary. The fault tree method is one of the essential tools for reliability analysis of complex and large scaled systems and is used in many kinds of literature.[6] Duan et al.[7] used fuzzy fault tree to analyze the reliability of flue gas turbine. Marquez et al.[8] conducted a qualitative analysis using the proposed

[1]State Key Laboratory of Mechanical Transmissions, Chongqing University, Chongqing, P.R. China
[2]Department of Mathematical Sciences, Durham University, Durham, United Kingdom

Corresponding author:
Yao Li, State Key Laboratory of Mechanical Transmissions, Chongqing University, Chongqing 400044, P.R. China.
Email: yao_li_@outlook.com

method based on fault tree analysis (FTA) and Binary Decision Diagrams. Zhang et al.[9] proposed the use of system grading and FTA to analyse the reliability of floating offshore wind turbines and discuss sequentially dependent failures and redundancy failures.

The redundancy design is quite common in WTs aiming at improving reliability, which is treated as the parallel system in much research.[10] In some studies,[11,12] the objective function of the optimization model is considered to be maximizing the system reliability. Li et al.[13] proposed a two-stage approach for solving multi-objective system reliability optimisation model that can obtain the best solution from the overall problem. Mahapatra and Roy[14] conducted the redundancy allocation for optimum reliability of series-parallel system, the aim of which is to maximise the system reliability. Ashraf et al.[15] established a fuzzy multi-objective optimization model for reliability-redundancy allocation problem (RRAP) using the multi-objective evolutionary algorithm and non-dominated sorting genetic algorithm with consideration of uncertainties in the parameters. Ardakan and Hamadani[16] studied reliability optimisations considering redundancy allocation problem and standby strategies. But redundancy components are treated as parallel system and independent of each other. Jahromi and Feizabadi[17] proposed a multi-objective evolutionary algorithm to solve the reliability model with the non-homogenous subsystem components, which takes the reliability and cost of the system as the objective function. Muhuri et al.[18] proposed a novel formulation of RRAP with fuzzy uncertainty. Huang et al.[19,20] developed a heuristic survival signature-based approach for reliability-redundancy allocation that reduces the dimension of the optimization problem and provides insight into system reliability-redundancy allocation.

In previous literature, all redundancy components are taken as parallel systems. This issue reduces the system reliability and leads to excessive system's reliability and much higher costs. Therefore, this article aims to study the influences of load-sharing of redundancy design on wind turbine reliability and reduce the whole system's cost. This remainder of this paper is organised as follows. The "Wind turbines" section introduces the development of wind power and the structure of wind turbines. The "Methodology" section proposes the load-sharing formulation with consideration of the correlation among components. FTA, RRAP and Markov chains are also presented in this section. The "Fault tree analysis of wind turbines" section shows the fault tree of the wind turbine and the procedures of using the fault tree to perform reliability assessment. Following this, this section also explores the method of obtaining the failure rates of load-sharing components. The "Reliability based redundancy allocation of the wind turbine" section gives MORRAP model and performs some runs at different time using GA. The "Reliability model and assessment of the wind turbine" section presents the results of three time-dependent reliability models to illustrate the performance of the load-sharing based reliability model compared with that of Markov-chains and parallel based reliability models. The "Conclusion" section summarizes some conclusions of this article.

Wind turbines

On the level of the global market, the size of the annual market has grown sharply year-on-year. By the end of 2018, the cumulative installed capacity climbed to 600 gigawatts (GW),[21] which is shown in Figure 1. However, with the growing number of wind turbines, the wind industry faces a lot of challenges. A number of WT components and assemblies are prone to failure, and it is difficult and expensive to repair and replace them.

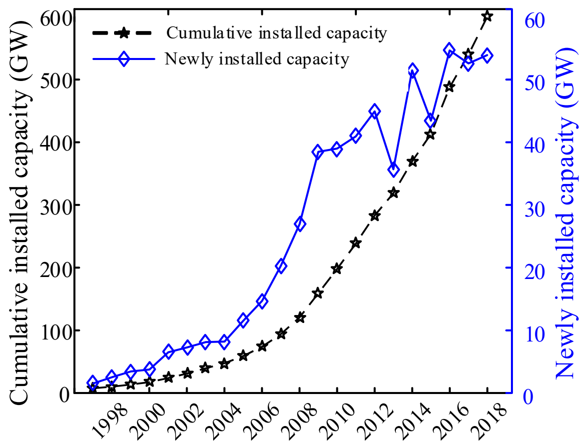


Figure 1. The global development of the wind power

The wind turbine structure is complex, and its operating condition is variable. A WT system consists of a software and communication unit, an electrical unit, a mechanical unit, a control unit and an auxiliary unit, which work together to guarantee the normal operation of the whole system. Any unit's fault may lead to the entire system's shutdown. Figure 2 shows the wind turbine system schematic. As can be seen from Figure 2, the entire wind turbine is a closed loop control system, and every subsystem has a close relationship with the others. Due to its complex structure and harsh operating environment, wind turbines are likely to suffer high failure rates. Therefore, high reliability is crucial to WTs. In reality, WT designers tend to use redundancy design to improve WT reliability. However, designers do not explore the effect of load-sharing and dependent failure in the reliability model of wind turbines.

Methodology

Notations

Load-sharing

In previous research, most redundancy components are treated as parallel units, which is not true in reality. For this reason, the quantification of the system's reliability of redundancy components is determined based on the assumption that when one redundancy component fails, the failure rates of other components does not change during the mission. However, this assumption is not

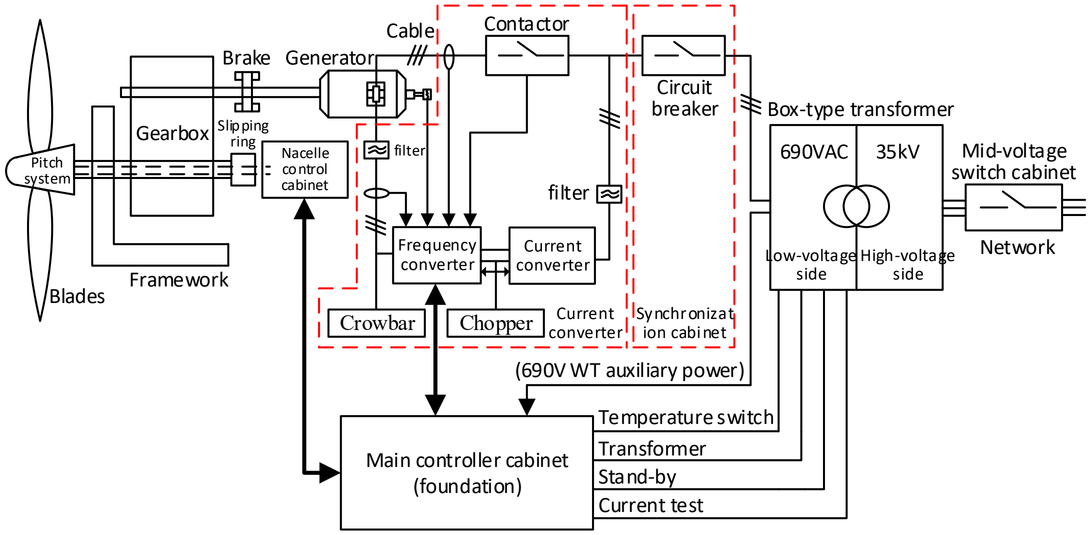


Figure 2. Wind turbine system schematic

$f_1(T)$	pdf of Weibull distribution of component 1 with parameters λ_1 and γ_1, when component 1 carries the load k_1L
$f_2(T)$	pdf of Weibull distribution of component 2 with parameters λ_2 and γ_2, when component 2 carries the load k_2L
$f_1^{'}(T)$	pdf of component 1 with parameters $\lambda_1^{'}$ and $\gamma_1^{'}$, when component 1 carries load L
$f_2^{'}(T)$	pdf of component 2 with parameters $\lambda_2^{'}$ and $\gamma_2^{'}$, when component 2 carries load L
$f_1^{''}(T-t_2)$	conditional pdf of component 1 after component 2 fails at time t_2 with $t_2 < T$
$f_2^{''}(T-t_1)$	conditional pdf of component 2 after component 1 fails at time t_1 with $t_1 < T$
$R_1^{''}(t-t_2)$	conditional reliability function of component 1 after component 2 fails at time t_2
$R_2^{''}(t-t_1)$	conditional reliability function of component 2 after component 1 fails at time t_1
$Q_1(t_1)$	probability of component 1 fails at time t_1
$Q_2(t_2)$	probability of component 2 fails at time t_2
$x_{i,j}$	number of jth components used in subsystem i
$c_{i,j}, s_{i,j}$	cost and space for the jth components in subsystem i
$\tilde{P}_i(t)$	individual state probabilities
$\tilde{\lambda}_i, \tilde{\gamma}_i$	scale and shape parameters of components taking sharing loads
λ_i, γ_i	shape and shape parameters of components taking total loads.
S	system-level constraint limit for space, set of s_i
N	set of subsystems without redundancy design
L	set of subsystems with redundancy design using load-sharing
M	set of subsystems' number of different components

correct in engineering. The remaining components will suffer higher failure rates due to the increased share of the load during the mission, which may reduce the system's reliability. For this reason, it is essential to take load-sharing into consideration in WT reliability analysis.

The two-unit load-sharing system with two-parameter Weibull distribution is used in this paper. The basic definitions are given here, and more detailed information is available in references by Liu[22] and Mattas[23].

The system's reliability function is given by:

$$R(t) = R_1(t)R_2(t) + Q_1(t_1)R_2(t_1)R_2^{''}(t-t_1) + Q_2(t_2)R_1(t_2)R_1^{''}(t-t_2) \tag{1}$$

The first term of equation (1) is the probability that component 1 and 2 complete their mission from 0 to t successfully with pdf's $f_1(t)$ and $f_2(t)$ respectively. It can be written as:

$$R_1(t) \cdot R_2(t) = e^{-(\lambda_1 \cdot t)^{\gamma_1}} \cdot e^{-(\lambda_2 \cdot t)^{\gamma_2}} \tag{2}$$

The second term of equation (1) is the probability that component 1 fails at $t_1 < t$ with pdf $f_1(T)$, and component 2 functions until t_1 with pdf $f_2(T)$ and then functions for the rest of the mission with pdf $f_2^{''}(T-t_1)$. It needs to use the equivalent-time technique to determine the $f_2^{''}(T-t_1)$ pdf or the conditional reliability function $R_2^{''}(T-t_1)$. The probability of the failure of component 2 when it carries the full load L is the same as the probability of the failure of component 2 when it carries the load k_2L. So

$$\int_0^{t_1} f_2(T)dT = \int_{t_1-t_{1e}}^{t_1} f_2^{'}(T-t_1)dT \tag{3}$$

Then through equation (3), the equivalent time t_{1e} can be obtained as follows:

$$t_{1e} = \frac{1}{\lambda_2^{'}} \cdot e^{(\lambda_2 \cdot t_1)^{\gamma_2/\gamma_2^{'}}} \tag{4}$$

The equivalent time t_{2e} can be obtained as well.

$$t_{2e} = \frac{1}{\lambda_1^{'}} \cdot e^{(\lambda_1 \cdot t_2)^{\gamma_1/\gamma_1^{'}}} \tag{5}$$

The second term of equation (1) can be written as follows:

$$Q_1(t_1)R_2(t_1)R_2^{''}(t-t_1) = \int_0^t \gamma_1 \cdot \lambda_1^{\gamma_1} \cdot t_1^{\gamma_1-1} \cdot e^{-\{(\lambda_1 \cdot t_1)^{\gamma_1} + [\lambda_2^{'}(t-t_1+\frac{1}{\lambda_2^{'}} \cdot e^{(\lambda_2 \cdot t_1)^{\gamma_2/\gamma_2^{'}}})]^{\gamma_2^{'}}\}} dt_1 \quad (6)$$

The third term of equation (1) can also be obtained in the same way, as follows:

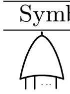
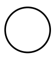
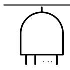
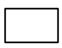
$$Q_2(t_2)R_1(t_2)R_1^{''}(t-t_2) = \int_0^t \gamma_2 \cdot \lambda_2^{\gamma_2} \cdot t_2^{\gamma_2-1} \cdot e^{-\{(\lambda_2 \cdot t_2)^{\gamma_2} + [\lambda_1^{'}(t-t_2+\frac{1}{\lambda_1^{'}} \cdot e^{(\lambda_1 \cdot t_2)^{\gamma_1/\gamma_1^{'}}})]^{\gamma_1^{'}}\}} dt_2 \quad (7)$$

By substituting the pdf's involved into equations (2), (6) and (7) and considering that $t_1, t_2 \in [0, t]$, the system's reliability for a mission of duration t can be obtained.

Fault tree analysis

FTA is a reliability analysis technique which starts from considering the system's failure modes and effects and is generally applicable to complex dynamic systems. There are two kinds of graphical symbols in the fault tree: gate symbols and event symbols. The gate symbols are used to represent and interconnect the various combinations of events. The most frequently used FT gate symbols are AND gate, OR gate, m-out-of-n voting gate and Priority AND gate. The event symbols are used to simplify the representation of a fault tree. Each symbol has its specific meaning. The basic symbolic descriptions are shown in Table 1. More details can be found in Marquez.[8]

Table 1. The symbolic descriptions

Symbol	Meaning	Sign	Meaning
	OR		Basic event
	AND		Resultant event

Multi-objective reliability-redundancy allocation

In most studies related to the reliability-redundancy allocation problem (RRAP), authors only focused on maximising the system reliability. Moreover, the redundancy components are treated as parallel systems, which leads to lower system reliability than normal. For this reason, the reliability of WTs in reality is too high, which brings a higher cost to WT manufactures and reduces their interest. Therefore, it is necessary to optimise the system reliability and the cost simultaneously, with consideration of the load-sharing. The proposed model for load-sharing based multi-objective reliability-redundancy allocation problem (MORRAP) is given as follows:

$$\begin{aligned} &Maximise: \; R(t;\mathrm{x}) = \prod_{i \in N} R_i^{sp}(t) \cdot \prod_{j \in L} R_j^{ls}(t) \\ &Minimise: \; C(t;\mathrm{x}) = \sum_{i=1}^{N} x_{ij} \cdot c_{ij} \end{aligned} \quad (8)$$

$$\begin{aligned} R_j^{ls}(t) = {} & e^{-(\lambda_j^1 \cdot t)^{\gamma_j^1}} \cdot e^{-(\lambda_j^2 \cdot t)^{\gamma_j^2}} + \int_0^t \gamma_j^1 \cdot (\lambda_j^1)^{\gamma_j^1} \\ & \cdot t_1{}^{(\gamma_j^1-1)} \cdot e^{-\{(\lambda_j^1 \cdot t_1)^{\gamma_j^1} + [\tilde{\lambda}_j^2(t-t_1+\frac{1}{\tilde{\lambda}_j^2} \cdot e^{(\lambda_j^2 \cdot t_1)^{\gamma_j^2/\tilde{\gamma}_j^2}})]^{\tilde{\gamma}_j^2}\}} dt_1 \\ & + \int_0^t \gamma_j^2 \cdot (\lambda_j^2)^{\gamma_j^2} \cdot t_2{}^{(\gamma_j^2-1)} \\ & \cdot e^{-\{(\lambda_j^2 \cdot t_2)^{\gamma_j^2} + [\tilde{\lambda}_j^1(t-t_2+\frac{1}{\tilde{\lambda}_j^1} \cdot e^{(\lambda_j^1 \cdot t_2)^{\gamma_j^1/\tilde{\gamma}_j^1}})]^{\tilde{\gamma}_j^1}\}} dt_2 \end{aligned} \quad (9)$$

Subject to:

$$\sum_{i=1}^{\mathrm{N}} \sum_{j=1}^{\mathrm{M}} x_{ij} \cdot s_{ij} \leq \mathrm{S} \quad (10)$$

$$1 \leq \sum_{i=1}^{N} x_{ij} \leq n_{max,j}, x_{ij} \in \{1, 2, \cdots, n_{max,i}\}$$

where $R_i^{sp}(t)$ represents the reliability of series-parallel subsystems at time t, $R_i^{ls}(t)$ means reliability of load-sharing subsystems at time t.

Reliability based Markov-chain model

The Markov-chain model has been used to deal with systems with multiple states and the interdependent components. In this article, it is used to obtain the dependent reliability in the series-parallel system. A parallel system with two components is taken as an example. There are three states for this system: both good, one good and one bad, or both bad. Figure 3 shows the state transition diagram of a two-component dependent failure reliability model. Every component has two models: normal and failed, so the number of the total models are 4 ($C_2^1 \cdot C_2^1$) for the two-component parallel system.

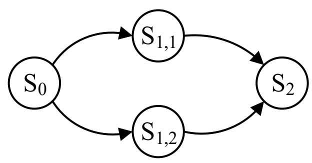


Figure 3. Reliability state transition diagram of dependent parallel system

Assumptions:

1. Both components are the same and have failure rates $\tilde{\lambda}_1(t)$ when both are working well.
2. Each component has failure rate $\tilde{\lambda}_2(t)$ if the other component fails. Here $\tilde{\lambda}_1(t) \leq \tilde{\lambda}_2(t)$.

The state transition-rate matrix is:[24]

$$T_R(t) = \begin{bmatrix} -2\tilde{\lambda}_1(t) & 0 & 0 & 0 \\ \tilde{\lambda}_1(t) & -\tilde{\lambda}_2(t) & 0 & 0 \\ \tilde{\lambda}_1(t) & 0 & -\tilde{\lambda}_2(t) & 0 \\ 0 & \tilde{\lambda}_2(t) & \tilde{\lambda}_2(t) & 0 \end{bmatrix} \quad (11)$$

The state probability of S_0 is solved directly by:

$$\tilde{P}_1(t) = \tilde{P}_1(t_0) \cdot e^{-2\int_{t_0}^{t} \lambda_1(\tau)d\tau} \quad (12)$$

The state probabilities of $S_{1,1}$ and $S_{1,2}$ are obtained by solving the homogeneous equation, the results are as follows:

$$\begin{aligned} \tilde{P}_j(t) =& \tilde{P}_1(t_0) \cdot [\int_{t_0}^{t} \lambda_1(u) \cdot e^{-2\int_{t_0}^{u} \lambda_1(\tau)d\tau + \int_{t_0}^{u} \lambda_2(\tau)d\tau} du] \\ & \cdot e^{-\int_{t_0}^{t} \lambda_2(\tau)d\tau} + \tilde{P}_j(t_0) \cdot e^{-\int_{t_0}^{t} \lambda_2(\tau)d\tau} \end{aligned} \quad (13)$$

where, $j = 2, 3$.

$$\tilde{P}_4(t) = 1 - \tilde{P}_1(t) - \tilde{P}_2(t) - \tilde{P}_3(t) \quad (14)$$

Then, the system reliability is obtained by:

$$R(t, t_0) = \tilde{P}_1(t) + \tilde{P}_2(t) + \tilde{P}_3(t) \quad (15)$$

Fault tree analysis of wind turbines

Fault tree of the WT

The research object in this paper is a doubly fed induction generator with four-point suspension produced by CSIC (Chongqing) Haizhuang Windpower Equipment Co., Ltd. The rotor diameter, the tower height and the rated power are 111m, 100m and 2.0 megawatt (MW), respectively. Some WT components are depicted in Figure 2. The WT is divided into six subsystems for a better FTA: critical rotor failure, critical gearbox failure, critical generator failure, nacelle failure, tower failure and yaw & pitch system failure, which are transformed into the FT of the entire WT shown in Figure 4. Each subsystem consists of hundreds of components. Without consideration of the redundancy design, the fault of any critical components will lead to the shutdown of the entire WT. The failure of some mechanical parts and auxiliary components does not cause shutdown of WTs. Therefore, all critical components that can lead to the shutdown are treated as series systems, and the redundancy designs are taken as parallel systems in the FT.

From the Figure 4, the minimal cut sets of subsystems are obtained as follows:

T: {E_1, E_2, E_3, E_4, E_5, E_6}
E_1: {1,2,3,4}, {2,3,4,5}
E_2: {6,7,9,10}, {6,7,9,11}, {6,8,9,10}, {6,8,9,11}
E_3: {12,15,16}, {13,15,16}, {14,15,16}
E_4: {17,18,19,21,23},{17,18,19,21,24},{17,18,19,22,23}, {17,18,19,22,24},{17,18,20,21,23},{17,18,20,21,24}, {17,18,20,22,23},{17,18,20,22,24}
E_5: {31,32,33}
E_6: {25,26,27,28,29,30}

where T and E_i $(i = 1, 2, \cdots, 6)$ are depicted in Figure 4.

The FT based system reliability can be obtained by:

$$\begin{aligned} R_{sys}(t) =& R_{E_1}(t) \cdot R_{E_2}(t) \cdot R_{E_3}(t) \cdot R_{E_4}(t) \\ & \cdot R_{E_5}(t) \cdot R_{E_6}(t) \end{aligned} \quad (16)$$

Failures rates of load-sharing

Most failure data monitored in reality is the mean time between failure that can not be used directly. Therefore, mean time between failure (MTBF) should be transformed into the parameters of Weibull distribution. A Weibull distribution $w(\lambda, \gamma)$ has two parameters: scale parameter λ and shape parameter γ. Hence, we can get the survivor function as follows:

$$S(t) = e^{-(\lambda \cdot t)^{\gamma}}, t > 0 \quad (17)$$

The rth moment $E(T^r)$ of the distribution is:

$$E(T^r) = \frac{\Gamma(1 + \frac{r}{\gamma})}{\lambda^r} \quad (18)$$

where, $\Gamma(k) = \int_0^{\infty} u^{k-1} e^{-u} du \;\; (k > 0)$ is the gamma function.

In this article, the r and γ are fixed at 1 and 2 respectively, and λ s.t. $E(T^r) = MTBF$. Then the scale parameter λ can be obtained by:

$$\ln \lambda = \frac{\ln(\int_0^{\infty} u^{k-1} e^{-u} du) - \ln(MTBF)}{r} \quad (19)$$

where, $k = 1 + \frac{r}{\gamma}$.

The failure rates for load-sharing is very different from that of parallel subsystems. For a load-sharing system with n components, the failure rate of ith component at time t is given by:

$$\lambda_i(t) = \frac{\lambda_s}{n_t} + \lambda_i \quad (20)$$

Where, n_t is the number of functioning components in load-sharing at time t, λ_s is the total failure rate related to the load that can be shared, λ_i is the further failure rate applying to component i.

According to equation (19) and equation (20), all related parameters of Weibull distribution can be obtained. The input parameters of the Weibull distribution of WT subsystems are shown in Table 2. The data of MTBF in this table is real maintenance records provided by CSIC (Chongqing) Haizhuang Windpower Equipment Co., Ltd.

Reliability based redundancy allocation of the wind turbine

Redundancy allocation is quite important for improving WT reliability and safety. In this section, the series-parallel system is composed of six subsystems and eight parallel systems, and a multi-objective optimisation using the genetic algorithm (GA) is explored. Component space and cost are constraints. The objective is to maximise the system reliability and minimise the system cost. The eight parallel subsystems are the gearbox cooling system (cooling fan n_1 and

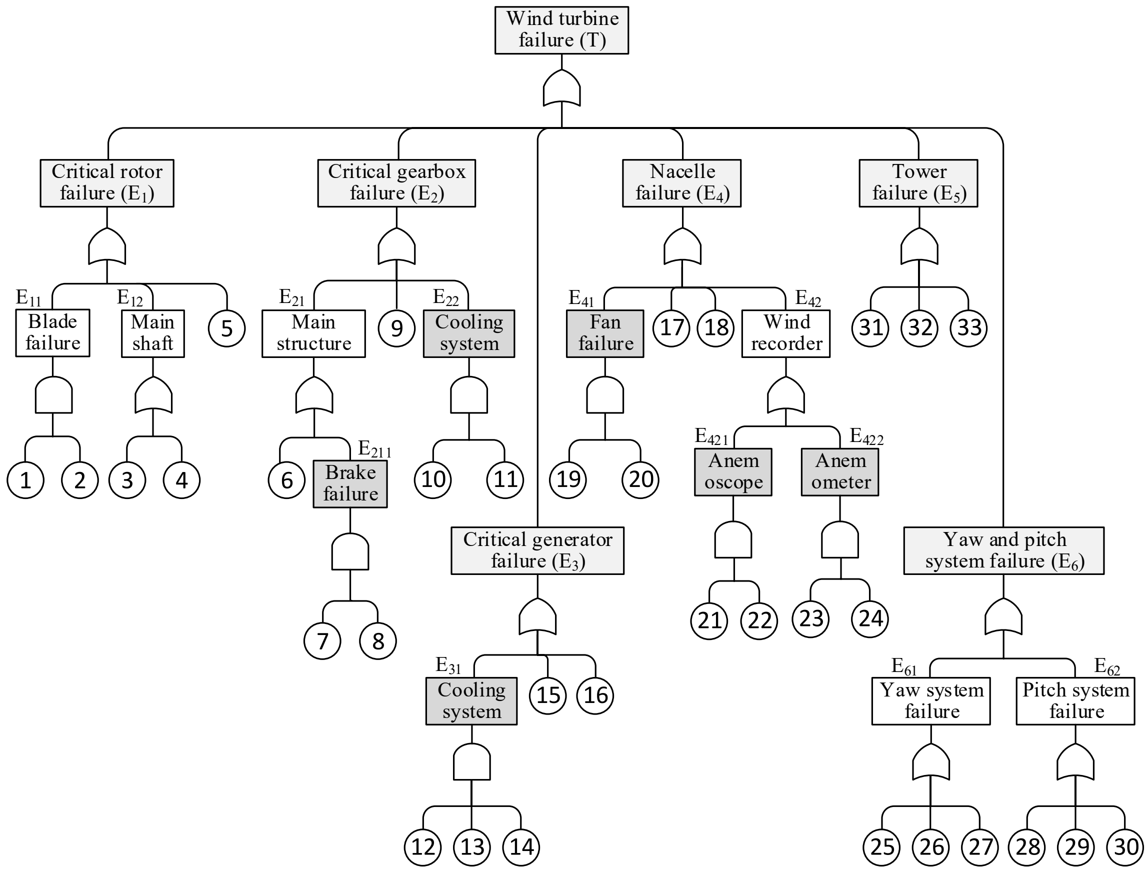


Figure 4. Fault tree for wind turbine

cooling pump n_2), generator cooling system (cooling fan n_3 and cooling pump n_4), nacelle cooling system (axial-flow fan n_5), wind vane (n_6), anemometer (n_7) and the hydraulic system (n_8) that are treated as the optimization variables represented by $[n_1, n_2, \cdots, n_8]$ respectively. The costs in this paper are relative values. The relative cost of each component is obtained based on the ratio of each component's cost and the total system cost.

Based on the MORRAP model with equations (8), (9) and (10), we performed some runs at different time t. One of the runs at time $t = 1$ year is shown in Figure 5. The results that consider integer constraints and non-integer constraints are shown in this figure. In engineering practice, the number of components or subsystems must be integer. As can be seen, the system reliability does not change when the total cost is between 240 and 360. However, if we want to improve the system reliability to 0.98, we have to spend much more on the system, which will increase the cost of WTs.

The results of reliability based redundancy-allocation optimization in the first year and second year are shown in Table 3 and Table 4. The "Cost Const." in the table means the cost constraints of redundancy design for the entire system. In Table 3, the best solution is selected as problem 22 ([1,1,1,1,1,2,1,2]) with the highest reliability (0.9792) and the acceptable cost (342.6). In addition, some other solutions can be obtained in different years: solution ([1,1,1,1,2,4,2,2]) in the second year with reliability (0.9199) and cost

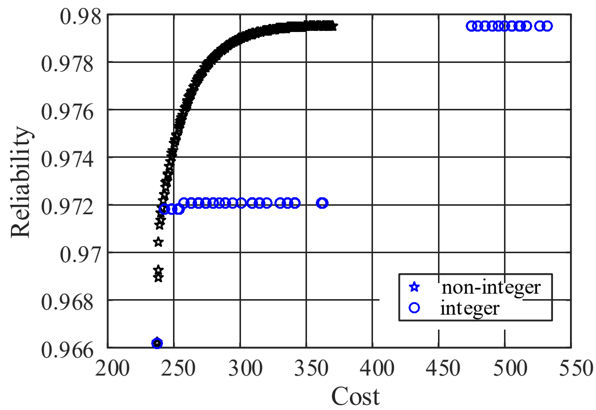


Figure 5. Runs of MORRAP

(374.2); solution ([1,1,1,1,1,3,2,2]) in the third year with reliability (0.8247) and cost (353.9); solution ([1,1,1,1,2,3,2,2]) in the fourth year with reliability (0.7090) and cost (368.9); solution ([1,1,1,1,2,2,1,2]) in the fifth year with reliability (0.5770) and cost (357.6). It is easy to find out that the redundancy design of the hydraulic system can greatly improve the WT reliability. To select the best solution, we propose to compare the system reliability and price-performance ratio (PPR) of different solutions. The results are shown in Table 5 and Figure 6. The PPR is calculated by equation (21). The results show that the first solution ([1,1,1,1,1,2,1,2]) is the best choice with the highest PPR and reliability at different time t. It means that the wind vane and the hydraulic system should be allocated more reliability to

Table 2. Parameters of fault tree of wind turbines

Basic event	Name	MTBF($10^4 h$)	γ	λ_s	n_t	$\lambda_i(t)$
1	Pitch driver failure	17.52	2	5.0584E-06	1	5.5584E-06
2	Pitch backup power failure	17.52	2	5.0584E-06	1	5.5584E-06
3	Main shaft	17.52	2	5.0584E-06	1	5.5584E-06
4	Main bearing	17.52	2	5.0584E-06	1	5.5584E-06
5	Hub	17.52	2	5.0584E-06	1	5.5584E-06
6	Elastic support	14.699	2	6.0292E-06	1	6.5292E-06
7	High-speed shaft brake	19.71	2	4.4963E-06	1	4.9963E-06
8	Pitch pneumatic brake	17.52	2	5.0584E-06	1	5.5584E-06
9	Lubrication system failure	1.623	2	4.3528E-05	1	4.8528E-05
10	Gearbox cooling fan failure	216.81	2	8.1751E-07	1	8.6751E-07
11	Gearbox cooling pump failure	144.54	2	1.4306E-06	1	1.9306E-06
12	Generator cooling fan1 failure	216.81	2	1.0219E-06	1	1.5219E-06
13	Generator cooling fan2 failure	86.724	2	1.0219E-06	0	1.5219E-06
14	Generator cooling pump failure	144.54	2	6.1314E-07	1	6.6314E-07
15	Lubrication system failure	43.362	2	2.0438E-06	1	2.5438E-06
16	Elastic support failure	144.54	2	6.1314E-07	1	6.6314E-07
17	Cantilever crane failure	61.946	2	1.4306E-06	1	1.9306E-06
18	Hydraulic system failure	2.955	2	0.3499E-04	2	0.2000E-04
19	Nacelle axial-flow fan1	17.52	2	5.0584E-06	1	5.5584E-06
20	Nacelle axial-flow fan2	17.52	2	5.0584E-06	0	5.5584E-06
21	Wind vane 1	4.20098	2	0.3499E-04	1	0.1555E-04
22	Wind vane 2	4.20098	2	0.3499E-04	1	0.1555E-04
23	Anemometer 1	216.81	2	4.0876E-07	1	4.5876E-07
24	Anemometer 2	216.81	2	4.0876E-07	0	4.5876E-07
25	Yaw bearing failure	17.52	2	5.0584E-06	1	5.5584E-06
26	Yaw driver failure	867.2425	2	1.0219E-07	1	1.5219E-07
27	Yaw drive failure	5.559	2	1.5942E-05	1	2.0942E-05
28	Pitch bearing failure	17.52	2	5.0584E-06	1	5.5584E-06
29	Pitch gearbox failure	144.54	2	6.1314E-07	1	6.1314E-07
30	Pitch pinion	17.52	2	5.0584E-06	1	5.5584E-06
31	Tower flange	17.52	2	5.0584E-06	1	5.5584E-06
32	Tower failure	17.52	2	5.0584E-06	1	5.5584E-06
33	Tower foundation	17.52	2	5.0584E-06	1	5.5584E-06

keep them working smoothly. Wind turbine designers need to pay more attention to the wind vane and the hydraulic system that are critical component and subsystem for WTs.

The price-performance ratio of solutions is obtained as follows:

$$PPR_i(t) = \frac{R_i^{sys}(t)}{C_i^{sys}(t)} \times C_s \tag{21}$$

where $R_i^{sys}(t)$ and $C_i^{sys}(t)$ are the system reliability and system cost of ith solution at time t respectively; C_s is the basic cost of the entire system that is equal to 350 in this article.

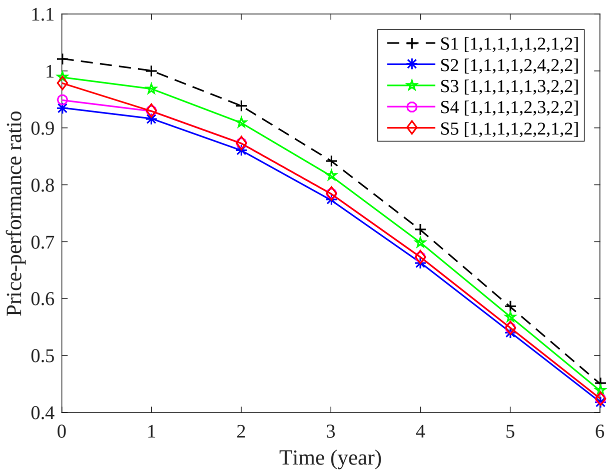


Figure 6. Performance analysis of redundancy-allocation solutions

Reliability model and assessment of the wind turbine

The best solution [1,1,1,1,1,2,1,2] is obtained from the previous section using reliability-based redundancy allocation. From this solution, we know that the wind vane and the hydraulic system need a redundancy design which adds one more component to the corresponding subsystem. The system's minimal cut set of the best solution of the WT is given below:

E_1: {1,2,3,4}, {2,3,4,5}
E_2: {6,7,9,10}, {6,7,9,11}, {6,8,9,10}, {6,8,9,11}
E_3: {13,15,16}, {14,15,16}
E_4: 2{17,18,20,21,24}, 2{17,18,20,22,24}
E_5: {31,32,33}
E_6: {25,26,27,28,29,30}

The system reliability of WT considering the factor i at time t can be obtained as follows:

$$\begin{aligned} R_{sys}^i(t) = & R_{E_1}(t) \cdot R_{E_2}(t) \cdot R_{E_3}(t) \cdot R_{E_4}^i(t) \\ & \cdot R_{E_5}(t) \cdot R_{E_6}(t) \end{aligned} \tag{22}$$

Table 3. Redundancy-allocation based reliability optimization results (T=1 year)

Problem	Cost const.	Solutions	Reliability	Cost
1	240	[1, 1, 1, 1, 1, 1, 1, 1]	0.9662	237.3
2	245	[1, 1, 1, 1, 1, 2, 1, 1]	0.9718	242.6
3	250	[1, 1, 1, 1, 1, 3, 1, 1]	0.9718	247.9
4	255	[1, 1, 1, 1, 1, 3, 2, 1]	0.9718	253.9
5	260	[1, 1, 1, 1, 2, 2, 1, 1]	0.9721	257.6
6	265	[1, 1, 1, 1, 2, 3, 1, 1]	0.9721	262.9
7	270	[1, 1, 1, 1, 2, 3, 2, 1]	0.9721	268.9
8	275	[1, 1, 1, 1, 2, 4, 2, 1]	0.9721	274.2
9	280	[1, 1, 1, 1, 2, 5, 2, 1]	0.9721	279.5
10	285	[1, 1, 1, 1, 3, 3, 2, 1]	0.9721	283.9
11	290	[1, 1, 1, 1, 3, 4, 2, 1]	0.9721	289.2
12	295	[1, 1, 1, 1, 3, 5, 2, 1]	0.9721	294.5
13	300	[1, 1, 1, 1, 3, 6, 2, 1]	0.9721	299.8
14	305	[1, 1, 1, 1, 3, 5, 3, 1]	0.9721	300.5
15	310	[2, 1, 1, 1, 3, 4, 2, 1]	0.9721	309.2
16	315	[2, 1, 1, 1, 3, 5, 2, 1]	0.9721	314.5
17	320	[2, 1, 1, 1, 3, 6, 2, 1]	0.9721	319.8
18	325	[2, 1, 1, 1, 3, 5, 3, 1]	0.9721	320.5
19	330	[2, 1, 1, 1, 3, 6, 3, 1]	0.9721	325.8
20	335	[2, 1, 2, 1, 3, 6, 4, 1]	0.9721	331.8
21	340	[2, 1, 2, 1, 3, 6, 4, 1]	0.9721	331.8
22	345	[1, 1, 1, 1, 1, 2, 1, 2]	0.9792	342.6
23	350	[2, 1, 2, 1, 3, 6, 3, 1]	0.9721	346.8
24	355	[2, 1, 2, 1, 3, 6, 3, 1]	0.9721	346.8
25	360	[2, 1, 2, 1, 4, 5, 3, 1]	0.9721	356.5
26	365	[2, 1, 2, 1, 4, 6, 3, 1]	0.9721	361.8
27	370	[1, 1, 1, 1, 2, 3, 2, 2]	0.9795	368.9
28	375	[2, 1, 2, 1, 4, 6, 5, 1]	0.9721	373.8
29	380	[2, 1, 2, 1, 4, 6, 3, 1]	0.9721	361.8
30	385	[1, 1, 1, 1, 3, 3, 2, 2]	0.9795	383.9
31	390	[2, 1, 1, 2, 4, 6, 4, 1]	0.9721	381.8
32	395	[1, 1, 1, 1, 3, 5, 2, 2]	0.9795	394.5

Table 4. Redundancy-allocation based reliability optimization results (T=2 years)

Problem	Cost const.	Solutions	Reliability	Cost
1	240	[1, 1, 1, 1, 1, 1, 1 ,1]	0.9662	237.3
2	245	[1, 1 ,1 ,1, 1, 2, 1, 1]	0.9718	242.6
3	250	[1, 1, 1, 1, 1, 3, 1, 1]	0.9718	247.9
4	255	[1, 1, 1, 1, 1, 3, 2, 1]	0.9718	253.9
5	260	[1, 1, 1, 1, 2, 2, 1, 1]	0.9721	257.6
6	265	[1, 1, 1, 1, 2, 3, 1, 1]	0.9721	262.9
7	270	[1, 1, 1, 1, 2, 3, 2, 1]	0.9721	268.9
8	275	[1, 1, 1, 1, 2, 4, 2, 1]	0.9721	274.2
9	280	[1, 1, 1, 1, 2, 5, 2, 1]	0.9721	279.5
10	285	[1, 1, 1, 1, 2, 6, 2 ,1]	0.9721	283.9
11	290	[1, 1, 1, 1, 3, 4, 2, 1]	0.9721	289.2
12	295	[1, 1, 1, 1, 3, 5, 2, 1]	0.9721	294.5
13	300	[1, 1, 1, 1, 3, 6, 2, 1]	0.8929	299.8
14	305	[1, 1, 1, 1, 3, 5, 3, 1]	0.8929	300.5
15	310	[2, 1, 1, 1, 3, 4, 2, 1]	0.8929	309.2
16	315	[2, 1, 1, 1, 3, 5, 2, 1]	0.8929	314.5
17	320	[2, 1, 1, 1, 3, 6, 2, 1]	0.8929	319.8
18	325	[2, 1, 1, 1, 3, 5, 3, 1]	0.8929	320.5
19	330	[2, 1, 1, 1, 3, 6, 3, 1]	0.8929	325.8
20	335	[2, 1, 2, 1, 3, 4, 2, 1]	0.8929	330.2
21	340	[2, 1, 2, 1, 3, 5, 2, 1]	0.8929	335.5
22	345	[2, 1, 2, 1, 3, 5, 3, 1]	0.8929	341.5
23	350	[2, 1, 2, 1, 3, 6, 3, 1]	0.8929	346.8
24	355	[2, 1, 2, 1, 3, 6, 3, 1]	0.8929	346.8
25	360	[2, 1, 2, 1, 4, 5, 3, 1]	0.8929	356.5
26	365	[2, 1, 2, 1, 4, 6, 3, 1]	0.8929	361.8
27	370	[2, 1, 2, 1, 4, 6, 3, 1]	0.8929	361.8
28	375	[1, 1, 1, 1, 2, 4, 2, 2]	0.9199	374.2
29	380	[2, 1, 2, 1, 4, 6, 4, 1]	0.8929	367.8
30	385	[1, 1, 1, 1, 2, 6, 2, 2]	0.9199	384.8
31	390	[2, 1, 3, 1, 4, 6, 3, 1]	0.8929	382.8
32	395	[1, 1, 1, 1, 3, 5, 2, 2]	0.9199	394.5

where $i \in$\{Par, LS, MC\}, which means parallel, load-sharing or Markov chain is taken into consideration of the reliability model.

According to the minimal cut set of the system and equation (22), the system's reliability with series-parallel subsystems is obtained shown in equation (23):

$$R_{sys}^{Par}(t) = R_{E_1}(t) \cdot R_{E_2}(t) \cdot R_{E_3}(t) \cdot R_{E_4}^{Par}(t) \cdot R_{E_5}(t) \cdot R_{E_6}(t) \tag{23}$$

where,

$$R_{E_1}(t) = R_{x_1}(t) \cdot R_{x_2}(t) \cdot R_{x_3}(t) \cdot R_{x_4}(t) + R_{x_2}(t) \cdot R_{x_3}(t) \cdot R_{x_4}(t) \cdot R_{x_5}(t) \tag{24}$$

$$\begin{aligned} R_{E_2}(t) = & R_{x_6}(t) \cdot R_{x_7}(t) \cdot R_{x_9}(t) \cdot R_{x_{10}}(t) \\ & + R_{x_6}(t) \cdot R_{x_7}(t) \cdot R_{x_9}(t) \cdot R_{x_{11}}(t) \\ & + R_{x_6}(t) \cdot R_{x_8}(t) \cdot R_{x_9}(t) \cdot R_{x_{10}}(t) \\ & + R_{x_6}(t) \cdot R_{x_8}(t) \cdot R_{x_9}(t) \cdot R_{x_{11}}(t) \end{aligned} \tag{25}$$

$$R_{E_3}(t) = R_{x_{13}}(t) \cdot R_{x_{15}}(t) \cdot R_{x_{16}}(t) + R_{x_{14}}(t) \cdot R_{x_{15}}(t) \cdot R_{x_{16}}(t) \tag{26}$$

$$R_{E_4}^{Par}(t) = 2R_{x_{17}}(t) \cdot R_{x_{18}}(t) \cdot R_{x_{20}}(t) \cdot R_{x_{24}}(t) \cdot (R_{x_{21}}(t) + R_{x_{22}}(t)) \tag{27}$$

$$R_{E_5}(t) = R_{x_{31}}(t) \cdot R_{x_{32}}(t) \cdot R_{x_{33}}(t) \tag{28}$$

$$R_{E_6}(t) = R_{x_{25}}(t) \cdot R_{x_{26}}(t) \cdot R_{x_{27}}(t) \cdot R_{x_{28}}(t) \cdot R_{x_{29}}(t) \cdot R_{x_{30}}(t) \tag{29}$$

The reliability of components following Weibull distribution is:

$$R_{x_i}(t) = e^{-(\lambda_i \cdot t)^{\gamma_i}}, t > 0 \tag{30}$$

Two subsystems take the load-sharing into consideration, which happens in the nacelle subsystem and the hydraulic subsystem. Therefore, according to equations (1), (6), (7) and (22), the load-sharing based reliability model of the WT is obtained as follows:

$$R_{sys}^{LS}(t) = R_{E_1}(t) \cdot R_{E_2}(t) \cdot R_{E_3}(t) \cdot R_{17}(t) \cdot R_{20}(t) \cdot R_{24}(t) \cdot R_{18}^{LS}(t) \cdot R_{E_{422}}^{LS}(t) \cdot R_{E_5}(t) \cdot R_{E_6}(t) \tag{31}$$

where $R_{18}^{LS}(t)$ and $R_{E_{422}}^{LS}(t)$ are the reliability of the hydraulic subsystem and the wind vane subsystem considering the load-sharing at time t, which are shown in equation (32) and (33) as follows:

$$\begin{aligned} R_{18}^{LS}(t) = & e^{-(\tilde{\lambda}_{18} \cdot t)^{\tilde{\gamma}_{18}}} e^{-(\lambda_{18} \cdot t)^{\gamma_{18}}} \\ & + \tilde{\gamma}_{18} \tilde{\lambda}_{18}^{\tilde{\gamma}_{18}} \int_0^t u^{\tilde{\gamma}_{18}-1} e^{-\{[\lambda_{18}'(t-u)]^{\gamma_{18}'} + (\tilde{\lambda}_{18} u)^{\tilde{\gamma}_{18}}\}} du \\ & + \gamma_{18} \lambda_{18}^{\gamma_{18}} \int_0^t u^{\gamma_{18}-1} e^{-\{[\tilde{\lambda}_{18}'(t-u)]^{\tilde{\gamma}_{18}'} + (\lambda_{18} u)^{\gamma_{18}}\}} du \end{aligned} \tag{32}$$

Table 5. Performance analysis of solutions

Time	Solution 1		Solution 2		Solution 3		Solution 4		Solution 5	
(year)	Reliability	PPR	Reliability	PPR	Reliability	PPR	Reliability	PPR	Reliability	PPR
1	0.9792	1.0004	0.9795	0.9162	0.9792	0.9685	0.9795	0.9293	0.9795	0.9293
2	0.9188	0.9387	0.9199	0.8604	0.9189	0.9088	0.9199	0.8727	0.9199	0.8727
3	0.8244	0.8422	0.8267	0.7732	0.8247	0.8156	0.8267	0.7843	0.8267	0.7843
4	0.7052	0.7205	0.7090	0.6631	0.7060	0.6982	0.7090	0.6727	0.7090	0.6727
5	0.5732	0.5856	0.5785	0.5411	0.5747	0.5683	0.5785	0.5488	0.5785	0.5488
6	0.4412	0.4507	0.4477	0.4187	0.4434	0.4385	0.4477	0.4247	0.4477	0.4247

$$
\begin{aligned}
R_{E_{422}}^{LS}(t) = & e^{-(\lambda_{21}\cdot t)^{\gamma_{21}}} e^{-(\lambda_{22}\cdot t)^{\gamma_{22}}} \\
& + \gamma_{21}\lambda_{21}^{\gamma_{21}} \int_0^t u^{\gamma_{21}-1} e^{-\{[\lambda_{22}^{'}(t-u)]^{\gamma_{22}^{'}} + (\lambda_{21}u)^{\gamma_{21}}\}} du \\
& + \gamma_{22}\lambda_{22}^{\gamma_{22}} \int_0^t u^{\gamma_{22}-1} e^{-\{[\lambda_{21}^{'}(t-u)]^{\gamma_{21}^{'}} + (\lambda_{22}u)^{\gamma_{22}}\}} du
\end{aligned} \tag{33}
$$

Submitting equations (32) and (33) to equation (31), the whole system's reliability with consideration of the load-sharing is obtained.

To investigate the proposed reliability model, we also study Markov-chain based reliability model in this article. According to equations (15) and (22), the system reliability model using the Markov-chain is obtained shown in equation (34). The reliability of the remaining five subsystems is calculated in the same way mentioned before. The difference focuses on the redundancy design of the hydraulic system and the wind vane system.

$$
\begin{aligned}
R_{sys}^{MC}(t) = & R_{E_1}(t) \cdot R_{E_2}(t) \cdot R_{E_3}(t) \cdot R_{17}(t) \cdot R_{20}(t) \\
& \cdot R_{24}(t) \cdot R_{18}^{MC}(t) \cdot R_{E_{422}}^{MC}(t) \cdot R_{E_5}(t) \cdot R_{E_6}(t)
\end{aligned} \tag{34}
$$

where $R_{18}^{MC}(t)$ and $R_{E_{422}}^{MC}(t)$ are the reliability of the hydraulic subsystem and the wind vane subsystem considering the Markov-chain at time t, which are shown in equations (35) and (36) as follows:

$$
\begin{aligned}
R_{18}^{MC}(t) = & \tilde{P}_1^{18}(t_0) e^{-2\int_{t_0}^t \tilde{\lambda}_{18}(\tau)d\tau} + 2\tilde{P}_1^{18}(t_0) \\
& \cdot \int_{t_0}^t \tilde{\lambda}_{18}(u) e^{-2\int_{t_0}^u \tilde{\lambda}_{18}(\tau)d\tau + \int_{t_0}^u \lambda_{18}(\tau)d\tau} du \\
& \cdot e^{-\int_{t_0}^t \lambda_{18}(\tau)d\tau} + \tilde{P}_2^{18}(t_0) e^{-\int_{t_0}^t \lambda_{18}(\tau)d\tau} \\
& + \tilde{P}_3^{18}(t_0) e^{-\int_{t_0}^t \lambda_{18}(\tau)d\tau}
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
R_{E_{422}}^{MC}(t) = & \tilde{P}_1^{23}(t_0) e^{-2\int_{t_0}^t \tilde{\lambda}_{23}(\tau)d\tau} + 2\tilde{P}_1^{23}(t_0) \\
& \cdot \int_{t_0}^t \tilde{\lambda}_{23}(u) e^{-2\int_{t_0}^u \tilde{\lambda}_{23}(\tau)d\tau + \int_{t_0}^u \lambda_{23}(\tau)d\tau} du \\
& \cdot e^{-\int_{t_0}^t \lambda_{23}(\tau)d\tau} + \tilde{P}_2^{23}(t_0) e^{-\int_{t_0}^t \lambda_{23}(\tau)d\tau} \\
& + \tilde{P}_3^{23}(t_0) e^{-\int_{t_0}^t \lambda_{23}(\tau)d\tau}
\end{aligned} \tag{36}
$$

Three kinds of reliability models for the WT are constructed in this article shown in equations (23), (31) and (34). The time-dependent reliability with different models is explored. Figure 7 shows the system reliability assessment using different reliability models. In this article, we compared the results of the proposed methods with those of the traditional methods. The results show that the load-sharing based reliability model can get the largest value of the system reliability with the same cost among the three reliability models. However, the reliability values of the Markov-chain and parallel based reliability models are second and third. The results are significantly different from those calculated when treating the redundancy design as parallel and Markov-chains subsystems. In the wind-power equipment industry, the redundant components and subsystems in WTs share the load and are dependent. Therefore, the redundancy design should not be treated as parallel systems, which would bring a significant error to the assessment of the system reliability. It is evident from Figure 7 that the system reliability would have been clearly underestimated if the components are taken as parallel subsystems, which means the components are assumed to be independent of each other. Using the proposed load-sharing based reliability model can contribute to more realistic estimates of the system reliability. The findings are directly in line with the reality that some components or subsystems are designed with much higher reliability than normal, which leads to the prohibitive cost of WTs.

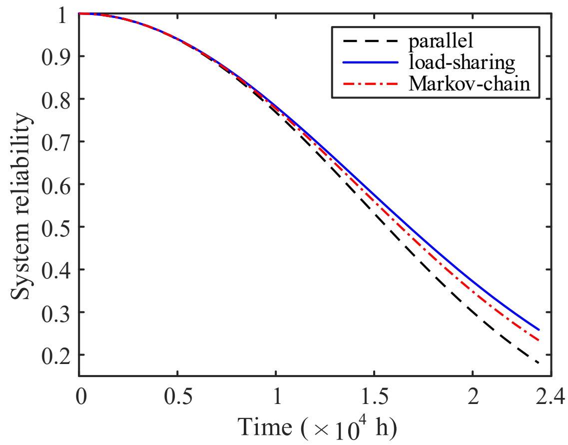


Figure 7. Reliability assessment with different reliability models

Conclusions

The wind turbine fault tree is established according to the failure mechanism and failure modes of the wind turbines. The load-sharing and Markov-chain methods are explored in this paper. The reliability and cost-based multi-objective reliability-redundancy allocation problem is proposed to get the optimal redundancy allocation, which can significantly reduce the system's cost and keep the system's reliability at a high level. Following this, the load-sharing based reliability model is proposed to analyse system reliability. To assess the performance of the proposed reliability model, we compare it with the results of the Markov-chain and parallel based reliability model. As we have argued in the reliability assessment section, the traditional methods may underestimate the system's reliability, which may lead to much waste on materials and the prohibitive cost. The findings show that the proposed reliability model can help obtain a more accurate value of the wind turbine reliability and reduce the system cost of the wind turbine. In summary, the load-sharing based reliability model can represent more realistic system reliability.

In future work, we will focus on the investigation of the effect of different reliability models on the lifetime of wind turbines.

Declaration of conflicting interests

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